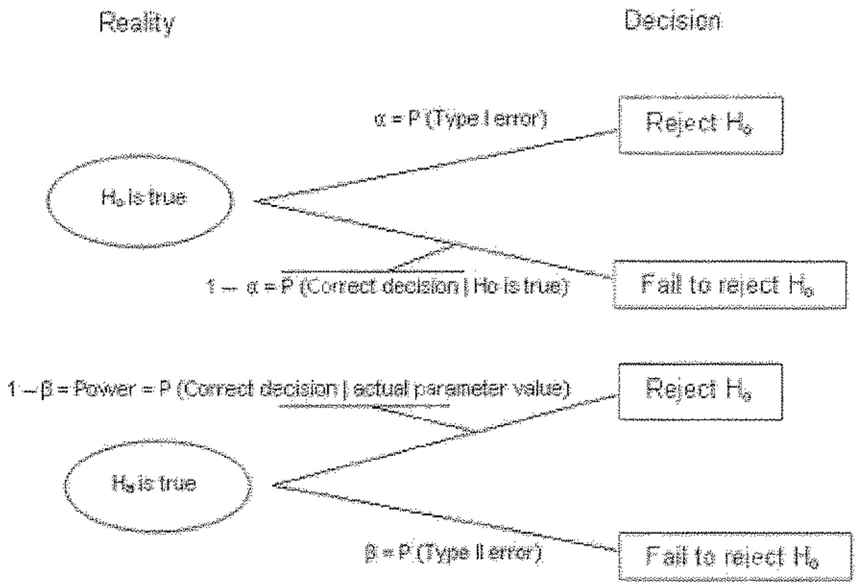
Chapter 21 Introduction to Power

The following tree diagram may help students appreciate the fact that α, β, and power are all conditional probabilities.



Power may be expressed in several different ways, and it might be worthwhile sharing more than one of them with your students, as one definition may "click" with a student where another does not. Here are a few different ways to describe what power is:

- Power is the probability of rejecting the null hypothesis when in fact it is false.
- Power is the probability of making a correct decision (to reject the null hypothesis) when the null hypothesis is false.
- Power is the probability that a test of significance will pick up on an effect that is present.
- Power is the probability that a test of significance will detect a deviation from the null hypothesis, should such a deviation exist.
- Power is the probability of avoiding a Type II error.

Calculating Power:

1. Given an α level, find z*.
2. Use z* to find the value of $\bar{x}$ on our H_0 curve.
3. Sketch where $\bar{x}$ is on the H_A curve. Find the new z on the second curve.
4. Find β and Power on the second curve.

How do changes in the following affect Power?

1. Sample size, n
2. α level
3. H_A

Example 10.18 Power for Testing Hypotheses About Proportions

A package delivery service advertises that at least 90% of all packages brought to its office by 9 A.M. for delivery in the same city are delivered by noon that day. Let π

denote the proportion of all such packages actually delivered by noon. The hypotheses of interest are

$$H_0\colon \pi = .9 \quad \text{versus} \quad H_a\colon \pi < .9$$

where the alternative hypothesis states that the company's claim is untrue. The value $\pi = .8$ represents a substantial departure from the company's claim. If the hypotheses are tested at level .01 using a sample of $n = 225$ packages, what is the probability that the departure from H_0 represented by this alternative value will go undetected?

At significance level .01, H_0 is rejected if P-value $\leq .01$. For the case of a lower-tailed test, this is the same as rejecting H_0 if

$$z = \frac{p - \mu_p}{\sigma_p} = \frac{p - .9}{\sqrt{\frac{(.9)(.1)}{225}}} = \frac{p - .9}{.02} \leq -2.33$$

(Because -2.33 captures a lower-tail z curve area of .01, the smallest 1% of all z values satisfy $z \leq -2.33$.) This inequality is equivalent to $p \leq .853$, so H_0 is *not* rejected if $p > .853$. When $\pi = .8$, p has approximately a normal distribution with

$$\mu_p = .8 \qquad \sigma_p = \sqrt{\frac{(.8)(.2)}{225}} = .0267$$

Then β is the probability of obtaining a sample proportion greater than .853, as illustrated in Figure 10.6.

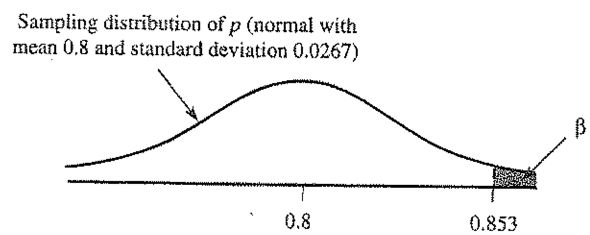


Figure 10.6 β when $\pi = .8$ in Example 10.18.

Converting to a z score results in

$$z = \frac{.853 - .8}{.0267} = 1.99$$

and Appendix Table 2 gives

$$\beta = 1 - .9767 = .0233$$

When $\pi = .8$ and a level .01 test is used, less than 3% of all samples of size $n = 225$ will result in a Type II error. The power of the test at $\pi = .8$ is $1 - .0233 = .9767$. This means that the probability of rejecting $H_0\colon \pi = .9$ in favor of $H_a\colon \pi < .9$ when π is really .8 is .9767, which is quite high.

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