

# Ch 21: More about Tests

AP Statistics – almost done with Unit  
5 (Ch 18 to Ch 22)

# Thinking about P Value

- P-value is a conditional probability
  - P of getting results given  $H_0$  is true
  - $P(\text{observed stat value or more extreme} \mid H_0)$
  - ALWAYS start with the Null Model!
  - NOT:
    - P that  $H_0$  is true
    - $P(H_0 \text{ is true} \mid \text{observed statistic value})$

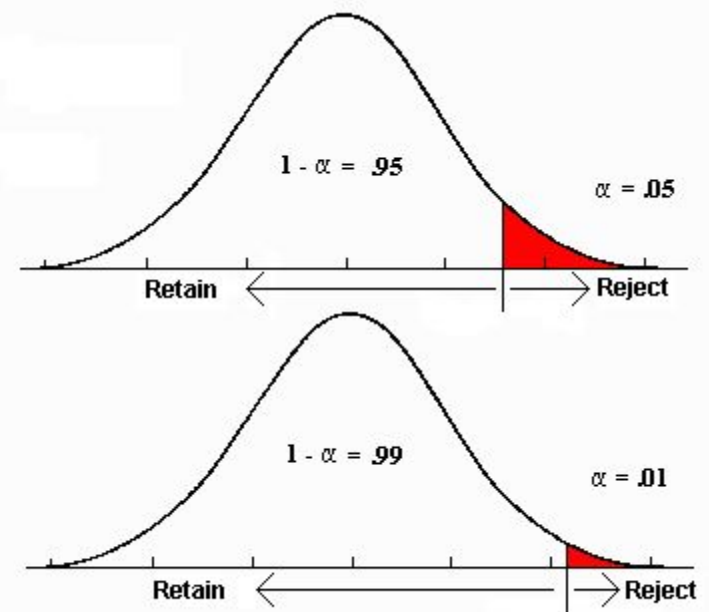
# Alpha Levels

- Small p-value?

Data seen are rare given the  $H_0$ .

- Rare enough?

- Alpha Level ( $\alpha$ ) = Significance Level: Threshold for P-value; if P-value is below it, we'll reject  $H_0$  and say results are statistically significant.
- Common alphas: 0.1, 0.05, 0.01 (e.g. “at the 0.1 level of significance”)
- Directly related to choice in CI. 90% CI means  $\alpha = 0.05$
- Pick an appropriate one for your study before collecting data



# Checklist for a One-Proportion Z-Test

- State  $H_0$  and  $H_a$
- Check conditions
- State you are using a One-Proportion Z-Test
- **State alpha**
- Write and draw your null model
- Calculate  $z$ , P-value
- Conclude
  - State assumption ( $H_0$ , null model)
  - Use P-value
  - **Use alpha and “statistically significant”**
  - State reject or fail to reject  $H_0$
  - Add CI based on  $H_0$  (if it helps add evidence) (state % interval, show  $z^*$  or mention 68/95/99.7 rule with 1/2/3 SD)– does value fall within or outside the interval, and what does this indicate?

# Practice: CI, alpha, conclusion re. H0

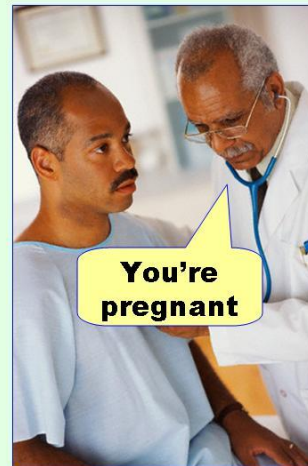
- A bank used ads compelling customers to set up payment plans, to avoid late fees. A 90% CI for the success rate is (0.29, 0.45). A prior method worked 30% of the time. Can you reject  $H_0: p = 30\%$  at  $\alpha = 0.05$ ? Draw a picture using the above information. You should not need to do any calculation to arrive at a conclusion.

# Type I and II Errors

- Recall we assume  $H_0$  is true, then calculate P-value and make a conclusion.
- We can make errors!
- Type I:  $H_0$  is true, but we mistakenly reject it.
  - False positive
- Type II:  $H_0$  is false, but we fail to reject it.
  - False negative
- P of a Type I error =  $\alpha$  ← we already know this
- P of a Type II error =  $\beta$  ←  $H_a$  is too broad to give us  $\beta$ ;  $\beta$  is your chance to give a specific effect you would like to see (e.g. a minimum cutoff)

		Reality	
		True	False
Measured/ Perceived	True	Correct 😊	Type I False Positive
	False	Type II False Negative	Correct 😊

**Type I error**  
(false positive)

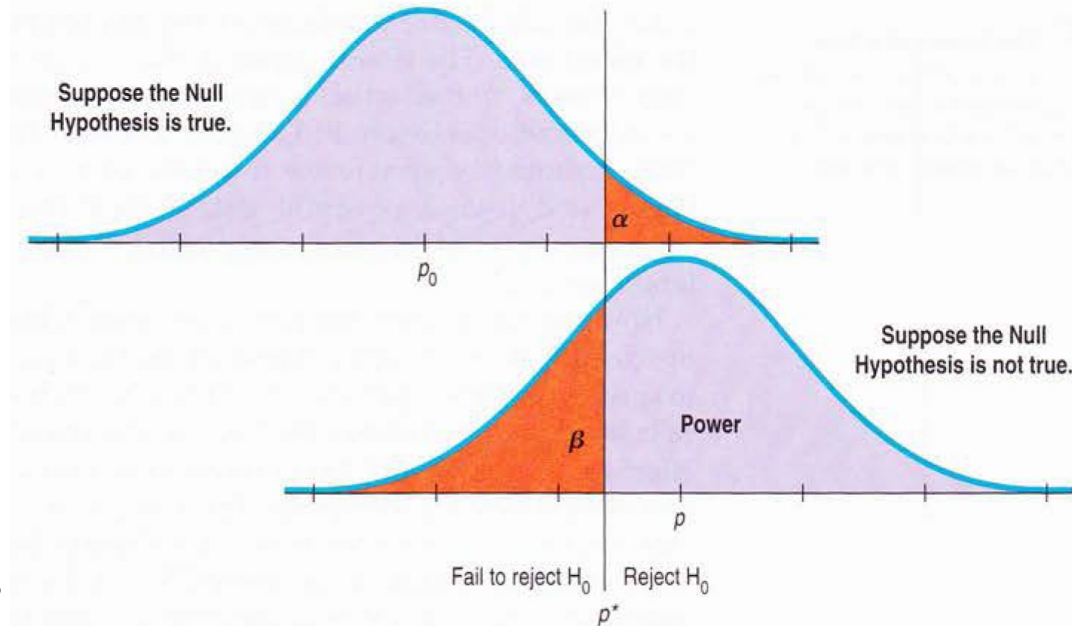


**Type II error**  
(false negative)



# Power

- Test's ability (P) to detect a false  $H_0$  = power of the test
  - This uses a “True Model” that is not the Null Model.
  - Ideally P is high
  - Power of test =  $1 - \beta$



Note: To calc Power, assume  $H_0$  is calculate P-value.)

How far is truth from  $H_0$ ? Distance btwn  $p_0$  and truth ( $p$ ) = effect size

Want more power? Increase one of these:

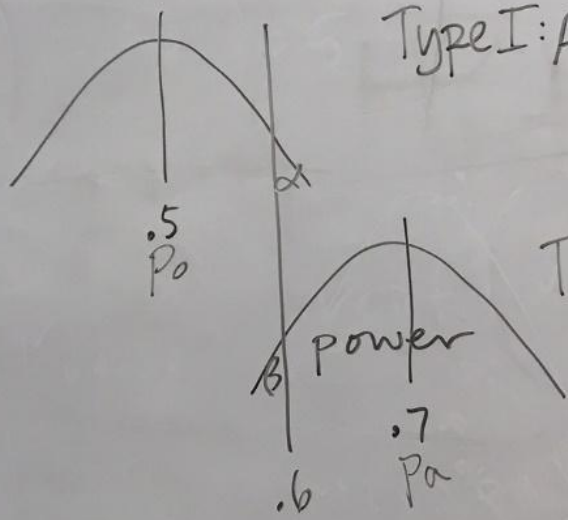
Increase  $n$ , effect size, alpha

# Power Example

The bank wants to spend more money on a new marketing strategy to get customers to make payments on time. It would like to see that this more expensive strategy produces a higher success rate than its 30% rate from before.

- What is a Type I error in this context, and what would be the consequences to the bank?
- What is a Type II error, and what would be the consequences to the bank?
- If the new strategy works, getting 60% of customers to pay on time, would the power of the test be higher or lower compared to a 32% pay off rate?



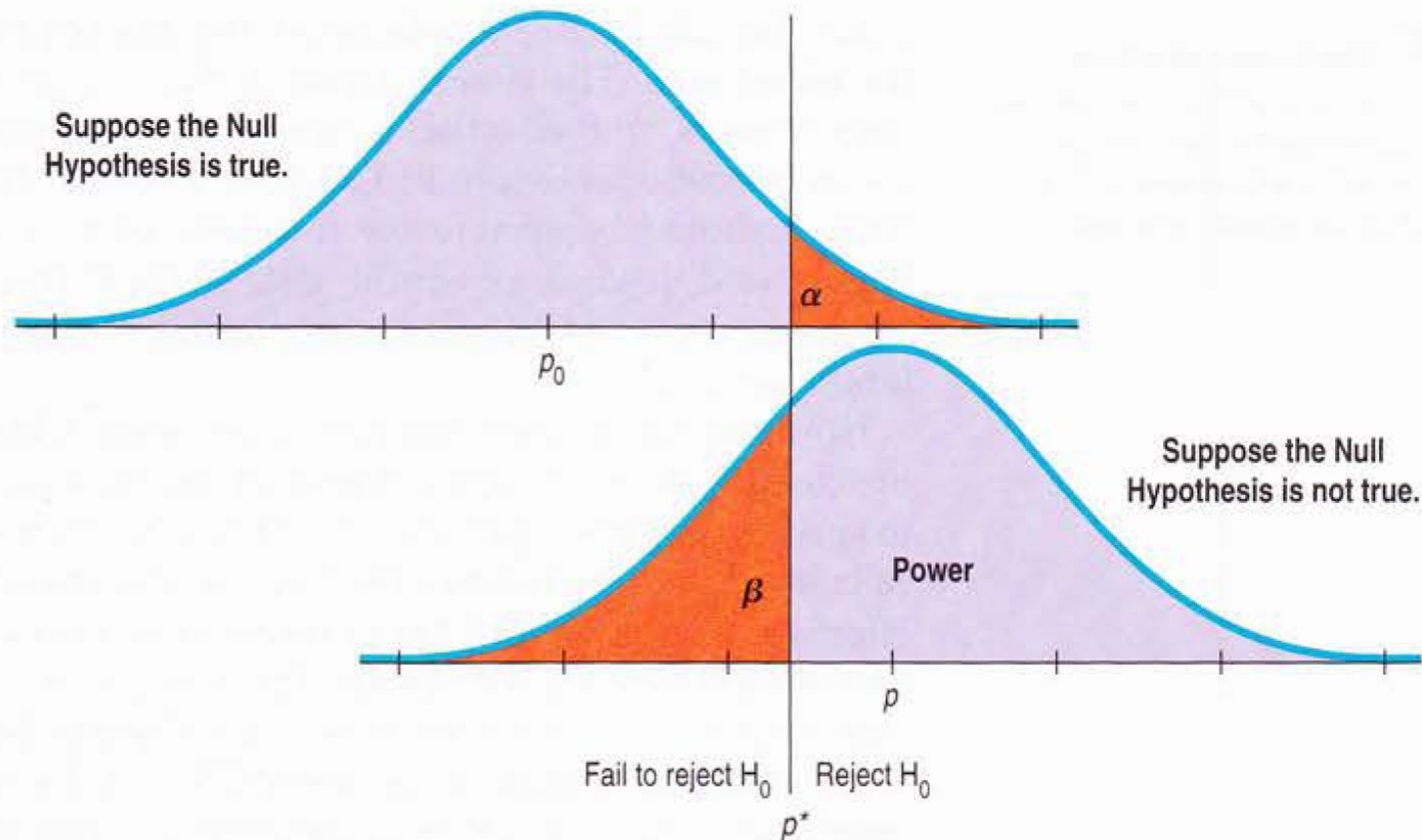


Type I: Assuming true  $\mu_p = .5$ .

$\bar{x}_p$  is  $.7$  and I reject  $H_0$ .

Type II: Assume true  $\mu_p = .7$ ,

$\bar{x}_p$  is  $.5$  and I fail to reject  $H_0$



- If  $\alpha$  increases, what happens to  $\beta$ ? Power?
- What if  $\alpha$  decreases?
- What if the true model's  $p$  increases? What happens to  $\alpha$ ,  $\beta$ , power?
- Define  $p^*$  using “critical value” in your sentence. Does it change when  $\beta$ , power, or the true model's  $p$  changes?

# Checklist for a One-Proportion Z-Test, More Thoroughly Presented

- State  $H_0$  and  $H_a$
- Check conditions
- State you are using a One-Proportion Z-Test
- State  $\alpha$
- Write and draw your null model
- Calculate  $z$ , P-value
- Conclude
  - State assumption ( $H_0$ , Null model)
  - Use P-value
  - Use  $\alpha$  and “statistically significant” and “type I” with a description
  - State reject or fail to reject  $H_0$
  - Add CI based on  $H_0$  (if it helps add evidence) (state % interval, show  $z^*$  or mention 68/95/99.7 rule with 1/2/3 SD)– does value fall within or outside the interval, and what does this indicate?
  - Describe  $p$  for an assumed true model,  $\beta$  (and “type II” with a description), power of the test.

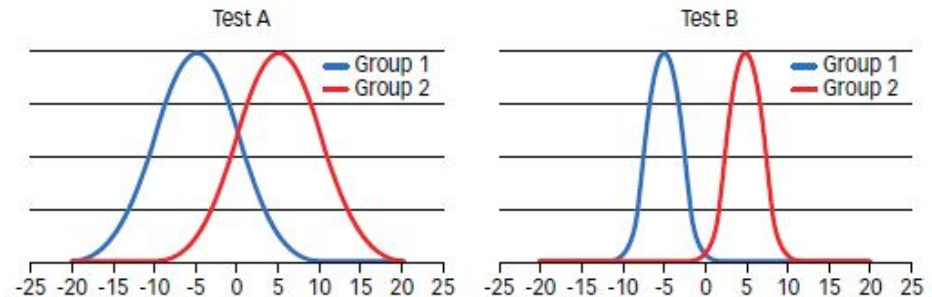
# Reducing both Type I and Type II Error

- Type I and Type II Error Ps can never be zero.
- Narrower variation decreases both errors.

What is parameter of variation for our models?

How do we decrease it?

**Comparing the difference between 2 distinct groups with low and high power** / FIGURE 1



Beware: Diminishing returns applies

\_\_\_\_\_ of a sampling dist. decreases as the square root of \_\_\_\_\_.